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Reflector Arrays

TOVE LARSEN

Abstract—**A method for the theoretical investigation of an arbitrary reflector array of the Van Atta type is described. The analysis is carried out for a reflector consisting of dipoles. Each pair of antenna elements is represented by an equivalent circuit. Mutual impedances and scattering by the dipoles are taken into account. The theory is illustrated by a numerical example of a linear reflector consisting of four half-wave dipoles. The result shows that the simple explanation of the action of a Van Atta reflector is not sufficient.**

I. Introduction

IN 1959, VAN ATTA [1] obtained a patent on a new type of electromagnetic reflector, which possesses the advantage that the reradiated field has a maximum back in the direction of arrival of the primary plane wave. The reflector consists of a number of antenna elements of which those placed symmetrically with respect to the geometrical center are connected in pairs by transmission lines of equal length. If the reflector has an odd number of elements, the center element is connected to a short-circuited transmission line, the length of which is half that of the other lines.

A qualitative and physical explanation of the method of operation of the reflector is given by Appel-Hansen [29].

Some experimental investigations have been carried out on this type of reflector, and many suggestions for improvement and utilization of the reflector have been made. However, a thorough theoretical treatment of the basic form of the Van Atta reflector does not seem to have been given. It is the purpose of this paper to describe a method for the theoretical and numerical treatment of an arbitrary Van Atta reflector, taking into account scattering by the antenna elements and mutual coupling between elements.

The investigations were initiated by Østfeldt [2], and the main results have been checked experimentally by Appel-Hansen [3], [29]. The paper is based on a more detailed report [4].

II. Survey of Previous Work

Van Atta's patent application was made in 1955. The idea of the reflector was already described in the literature by Bloch [5] in 1956, but the main bulk of literature appeared after Van Atta obtained his patent in 1959. Most of the papers have concerned the use of auxiliary active equipment in connection with a Van Atta reflector, or the various applications of this type of reflector. Only the experimental work of Sharp, Fusca, and Diab [6]–[8] on an electromagnetic dipole reflector, and that of Walther [9] on an acoustic horn reflector seems to be of basic character. No thorough theoretical treatment of the properties of a simple Van Atta reflector has been found in the literature.

The work of Sharp, Fusca, and Diab describes the construction and examination of a two-dimensional array, consisting of 16 dipoles mounted one-quarter of a wavelength above a conducting plane. A simplified theory for this array was given, and several diagrams of the backscattering cross section as a function of the angle of incidence were presented. These diagrams were measured for various polarizations and frequencies. For all the examples shown, the array had a high backscattering cross section over a wider range of angles of incidence than the normal plate reflector of the same size.

Walther constructed an acoustic Van Atta array, consisting of 36 conical horns arranged on a flat surface. Measured back-scattering cross section diagrams showed high reflection over a range of angles of incidence greater than that for a conventional corner reflector of the same size of aperture.

Two short notes tend to give an analytical treatment of the Van Atta reflector. Bauer [10] has taken into account scattering by the antenna elements, but, as the main purpose of his note is to suggest amplitude modulation of the reflector, he does not mention the influence of other parameters on the properties of the basic reflector. Kurss and Kahn [11] give a short general theory for the effect of lossless interconnection of elements in a passive array. However, without justification, they anticipate that the current distribution causing reradiation has the same amplitude on all elements and a uniform phase progression, which should cause a reradiated plane wave directed either back in the direction of the incoming wave (Van Atta principle) or in the direction of reflection.

The idea of making the Van Atta reflector active by inserting active components in the transmission lines was first proposed by Bauer [10], who suggested the introduction of modulated phase shifters in the transmission lines. The insertion of amplifiers in the transmission line was suggested by Hansen [12]. Mechanical modulation by means of a cavity resonator was proposed by Wanselow [13]. Davies [14] discussed the effect of amplifiers in the transmission lines, as well as the features of a circular Van Atta array. Further, he proposed a way of varying the angle of retransmission by introducing a frequency change in the delay paths. Later, a discussion [15] of Davies' work was published

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together with the description of a related experimental investigation which was carried out by Whithers [16].

One proposed application of the Van Atta reflector is to satellite communication; both passive, semipassive, and active systems have been discussed, especially by Ryerson [17], [18], Hansen [12], and Kaiser and Kay [19]. Other possible applications are to navigational aids; for example, Van Atta reflectors may be used to enhance the reflection from radar targets on small ships and airplanes (discussed by Davies [14]). Fusca [7] suggested an ECM system based on the idea that an artificial enhancement of radar return signals would confuse enemies. Bauer [10] suggested a passive IFF system based on coded modulation of the reflector. Bauer's idea was criticized by Bahret [20].

The Van Atta reflector is one of the most simple forms of the group of adaptive or self-phasing arrays which has received a great deal of attention in recent literature [21], [22]. Most of these systems are active and rather complicated; the applications seem to be numerous.

III. Theoretical Investigation

In what follows, an analysis will be given of an arbitrary Van Atta reflector consisting of dipoles. The method of calculation may be used for reflectors with other types of antenna elements. However, as different antennas have different properties, the results will depend greatly on the type of antennas used.

Dipoles were chosen partly because they were used in the experimental investigations [6]–[8] described previously, and partly because the characteristics of dipoles are very well known. Also, dipole reflectors have an advantage in mechanical construction and space requirements compared to, for example, horn reflectors.

The system to be examined is shown in Fig. 1. The dipole elements lie on, and are parallel to, a (mathematical) surface called the reflector surface. The incident field is a plane wave.

A reference plane is introduced as the plane tangential to the reflector surface at a conveniently selected point O. Further, a rectangular coordinate system, with O as origin and z-direction perpendicular to the reference plane, is introduced. The angles of incidence of the primary plane wave in this system are ϕ_i and θ_i, and the angle between the electric vector $\bar{E}=E_0\hat{E}_0$ of the incident wave and the plane of incidence is v. [Unit vectors are denoted by carets ( )]. The nth dipole element has the coordinates (x_n, y_n, z_n) and makes the angles ϕ_n and θ_n with the x-axis and z-axis, respectively.

The calculations will proceed as follows. An equivalent circuit for each pair of antenna elements is found. From these, a system of equations is obtained for the unknown currents. These equations may be solved by an electronic computer, and, from the currents found, the reradiated field may be obtained by using ordinary antenna array theory (only applicable for reflectors with all elements parallel) or a similar theory. The reradiation will be described by the differential scattering cross section.

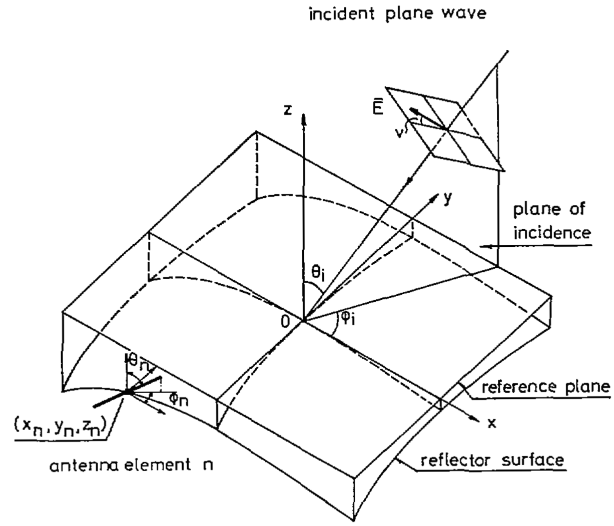


Fig. 1. Reflector surface with coordinate systems.

A. *Equivalent Circuit for a Pair*

An equivalent circuit for a connected pair is shown in Fig. 2(a), and for an element connected to a short-circuited line in Fig. 2(b). The equivalent circuit for the antenna elements themselves was described by Kraus [23], and was discussed more recently by Harrington [24]. According to Harrington, this diagram is valid if the scattered field of the open-circuited antenna is small compared to that of the terminated antenna. From the curves calculated by Hu [25], this is found to be true for half-wave, and smaller, dipoles.

In the antenna circuits, V_n is the open-circuit voltage induced in the antennas, given by

$$V_n = \bar{E}\cdot\bar{L}_{\text{eff}}, \tag{1}$$

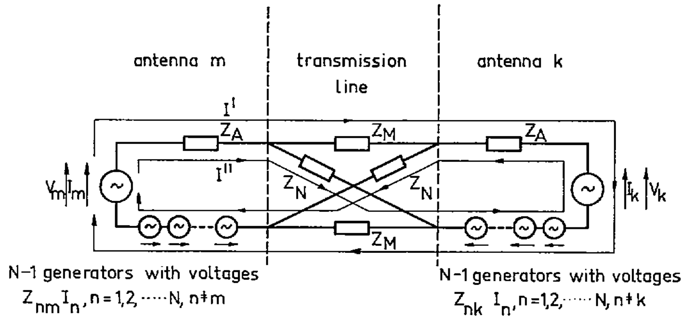
where $\bar{E}$ is the electric field strength at the position of the element, and $\bar{L}_{\text{eff}}$ the effective length [26] of the element. For dipoles in the coordinate system chosen, we have

$$\bar{L}_{\text{eff}} = \frac{\lambda}{\pi}\,\frac{\cos\left(\dfrac{\pi}{\lambda}L\cos u\right)-\cos\dfrac{\pi}{\lambda}L}{\sin\left(\dfrac{\pi}{\lambda}L\right)\sin u}(\sin\theta_n\cos\phi_n\hat{x}$$
$$+\sin\theta_n\sin\phi_n\hat{y}+\cos\theta_n\hat{z}), \tag{2}$$

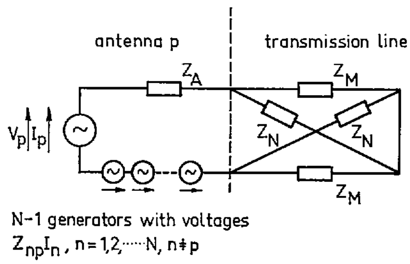
where L is the length of the dipole, and u the angle between the direction of the dipole and the direction of propagation of the primary plane wave:

$$\cos u = \sin\theta_n\sin\theta_i\cos(\phi_n-\phi_i)+\cos\theta_n\cos\theta_i. \tag{3}$$

The impedance Z_A is fhe self-impedance of an antenna element, and Z_{nk} is the mutual impedance between the nth and the kth element. The influence of the mutual impedances is taken into account by the introduction of equivalent generators with voltages $Z_{nk}I_k$, as shown in Figs. 2(a) and 2(b). This is in accordance with the definition of the mutual impedance between two antennas as the ratio between the open cir-



(a)



(b)

Fig. 2. (a) Equivalent circuit of two connected array elements. (b) Equivalent circuit of array element connected to short-circuited transmission line.

cuit voltage induced across the terminals of one antenna, due to a current flowing in the other antenna, and this current.

Each transmission line is represented by an X-circuit, the most general equivalent circuit of a twoport. The values of the two impedances Z_M and Z_N are

$$Z_M = -iZ_0 \tan\frac{ka}{2} \tag{4}$$

$$Z_N = iZ_0 \frac{1}{\tan\frac{ka}{2}}, \tag{5}$$

where Z_0 is the characteristic impedance, k the propagation constant, and a the length of the transmission lines. The time factor is $e^{-i\omega t}$.

Other equivalent circuits could be used as well, but the X-circuit shows directly what happens when the transmission line is an odd number of half-wavelengths or an integral number of wavelengths. For $a=(2p+1)\lambda/2$ (p being an integer), we get a cross connection, and for $a=p\lambda$, we get a direct connection. In these special cases, the currents of the two mates will be equal with the same, or opposite sign, respectively.

B. Equations for Determination of the Antenna Currents

Let us consider a reflector array consisting of N elements numbered n, $1 \le n \le N$. The elements are mutually connected with transmission lines, and if N is odd, one of the elements is connected to a short-circuited transmission line. In this way, we get S pairs, numbered s, $1 \le s \le S$. For N even, $S=N/2$, and for N odd, $S=(N-1)/2$. The sth pair consists of the sth and the $(N+1-s)$th element connected with a transmission line. For N odd, we get an additional system, numbered $s=S+1$, consisting of the $(N+1)/2$th element connected to a short circuit transmission line.

For a typical pair, we introduce the mesh currents I' and I'' [Fig. 2(a)], and have

$$I_m = I' + I'' \tag{6}$$

$$I_k = -I' + I''. \tag{7}$$

From the mesh in which I' flows, we derive

$$V_m - V_k = (Z_A + Z_M - Z_{mk})(I_M - I_k) + \sum_{n=1, n\neq m,k}^{N} I_n(Z_{nm} - Z_{nk}) \tag{8}$$

and from the mesh in which I'' flows we obtain

$$V_m + V_k = (Z_A + Z_N + Z_{mk})(I_m + I_k) + \sum_{n=1, n\neq m,k}^{N} I_n(Z_{nm} + Z_{nk}). \tag{9}$$

Similar equations have been found by Tai [27] by another method.

From Fig. 2(b) we obtain directly

$$V_p = I_p\left(Z_A + 2\frac{Z_N Z_M}{Z_N + Z_M}\right) + \sum_{n=1, n\neq p}^{N} I_n Z_{np}. \tag{10}$$

In order to operate with dimensionless quantities, we normalize the voltages, currents, and impedances by dividing by the normalization factors $V_0 = E_0\lambda/\pi$, $I_0 = E_0\lambda/\pi Z_0$, and Z_0.

Further, we introduce the values (4) and (5) for Z_M and Z_N. Using lower case letters for normalized quantities, we finally obtain

$$(v_m - v_k)\cos\frac{ka}{2} = \left(-i\sin\frac{ka}{2} + (z_A - z_{mk})\cos\frac{ka}{2}\right) \cdot (i_m - i_k) + \sum_{n=1, n\neq m,k}^{N} i_n(z_{nm} - z_{nk})\cos\frac{ka}{2}, \tag{11}$$

$$(v_m + v_k)\sin\frac{ka}{2} = \left(i\cos\frac{ka}{2} + (z_A + z_{mk})\sin\frac{ka}{2}\right) \cdot (i_m + i_k) + \sum_{n=1, n\neq m,k}^{N} i_n(z_{nm} + z_{nk})\sin\frac{ka}{2}, \tag{12}$$

$$v_p \cos ka = i_p(z_A \cos ka - i \sin ka) + \sum_{n=1, n\neq p}^{N} i_n z_{np} \cos ka. \tag{13}$$

This notation has been introduced in order to avoid the infinite values of Z_N and Z_M. From the equations it is seen that we get the following results for the special lengths of the transmission lines, as mentioned above.

$$a = (2p+1)\frac{\lambda}{2} \qquad i_m = i_k \tag{14}$$

$$a = p\lambda \qquad i_m = -i_k \tag{15}$$

$$a = (2p+1)\frac{\lambda}{4} \qquad i_{(N+1)/2} = 0, \tag{16}$$

where p is an integer.

Now the following N-dimensional complex matrix equation in the N unknown currents is found.

$$\begin{bmatrix} \bar{v}_1 \\ \bar{v}_2 \\ \vdots \\ \bar{v}_S \\ \overset{0}{v}_{S+1} \\ \overset{+}{v}_1 \\ \overset{+}{v}_2 \\ \vdots \\ \overset{+}{v}_S \end{bmatrix} = \begin{bmatrix} \bar{z}_{1,1} & \bar{z}_{1,2} & \cdots & \bar{z}_{1,S+1} & \cdots & \bar{z}_{1,N} \\ \bar{z}_{2,1} & \bar{z}_{2,2} & \cdots & \bar{z}_{2,S+1} & \cdots & \bar{z}_{2,N} \\ \vdots & \vdots & & \vdots & & \vdots \\ \bar{z}_{S,1} & \bar{z}_{S,2} & \cdots & \bar{z}_{S,S+1} & \cdots & \bar{z}_{S,N} \\ \overset{0}{z}_{S+1,1} & \overset{0}{z}_{S+1,2} & \cdots & \overset{0}{z}_{S+1,S+1} & \cdots & \overset{0}{z}_{S+1,N} \\ \overset{+}{z}_{1,1} & \overset{+}{z}_{1,2} & \cdots & \overset{+}{z}_{1,S+1} & \cdots & \overset{+}{z}_{1,N} \\ \overset{+}{z}_{2,1} & \overset{+}{z}_{2,2} & \cdots & \overset{+}{z}_{2,S+1} & \cdots & \overset{+}{z}_{2,N} \\ \vdots & \vdots & & \vdots & & \vdots \\ \overset{+}{z}_{S,1} & \overset{+}{z}_{S,2} & \cdots & \overset{+}{z}_{S,S+1} & \cdots & \overset{+}{z}_{S,N} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ i_N \end{bmatrix} \tag{17}$$

where

$$\bar{v}_s = (v_s - v_{N+1-s})\cos\frac{ka}{2} \tag{18}$$

$$\overset{+}{v}_s = (v_s + v_{N+1-s})\sin\frac{ka}{2} \tag{19}$$

$$\bar{z}_{s,s} = -\bar{z}_{s,N+1-s} = -i\sin\frac{ka}{2} + (z_A - z_{s,N+1-s})\cos\frac{ka}{2} \tag{20}$$

$$\bar{z}_{s,n} = \bar{z}_{n,s} = -\bar{z}_{N+1-s,n} = (z_{n,s} - z_{n,N+1-s})\cos\frac{ka}{2} \qquad n \neq s,\ N+1-s \tag{21}$$

$$\overset{+}{z}_{s,s} = \overset{+}{z}_{s,N+1-s} = i\cos\frac{ka}{2} + (z_A + z_{s,N+1-s})\sin\frac{ka}{2} \tag{22}$$

$$\overset{+}{z}_{s,n} = \overset{+}{z}_{n,s} = \overset{+}{z}_{N+1-s,n} = (z_{n,s} + z_{n,N+1-s})\sin\frac{ka}{2} \qquad n \neq s,\ N+1-s. \tag{23}$$

For N odd we get the additional quantities

$$\overset{0}{v}_{S+1} = v_{(N-1)/2}\cos ka \tag{24}$$

$$\overset{0}{z}_{S+1,S+1} = z_A\cos ka - i\sin ka \tag{25}$$

$$\overset{0}{z}_{S+1,n} = \overset{0}{z}_{S+1,N+1-n} = z_{(N-1)/2,n}\cos ka \qquad n \neq S+1 \tag{26}$$

$$\bar{z}_{s,S+1} = 0 \tag{27}$$

$$\overset{+}{z}_{s,S+1} = 2z_{s,(N-1)/2}\sin\frac{ka}{2}\,. \tag{28}$$

C. Scattering Cross Section of Reflector

When the current in each antenna element is found by an electronic computer solution of the matrix equation (17), the reradiated field is determined by

$$\bar{E} = \bar{E}_{\text{ref}}(R, \theta, \phi)\cdot G(\theta, \phi), \tag{29}$$

where

$$\bar{E}_{\text{ref}} = \zeta_0 I_0 \frac{e^{ik_0R}}{R}\bar{F}(\theta, \phi) \tag{30}$$

is the field radiated by a reference antenna (i.e., an antenna identical with and oriented similarly to the other antennas, placed at the reference point O), and

$$G(\theta, \phi) = \sum_{n=1}^{N} i_n e^{-ik_0\bar{r}_n\cdot\hat{R}} \tag{31}$$

is the array characteristic.

Here, ζ_0 is the intrinsic impedance of free space, k_0 the free-space propagation constant, $\bar{F}$ a dimensionless vector function characteristic of the type of antennas used, $\bar{r}_n$ the radius vector from O to the nth antenna, and $\hat{R}$ the unit vector directed from O to the field point.

A convenient quantity for describing the scattering from an object is the differential scattering cross section, which, for plane wave incidence, is defined by

$$\sigma(\theta, \phi) = 4\pi R^2 \frac{|\bar{S}_r(\theta, \phi)\cdot\hat{n}_r|}{|\bar{S}_i(\theta_i, \phi_i)\cdot\hat{n}_i|}, \tag{32}$$

where $\bar{S}_r$ and $\bar{S}_i$ are the Poynting vectors of the reradiated field at the distance R and the incoming field, respectively, and $\hat{n}_r$ and $\hat{n}_i$ are unit vectors in the direction of the reradiated and the incoming field.

For the incidence plane wave we have

$$|\bar{S}_i\cdot\hat{n}_i| = \frac{1}{2}\frac{E_0^2}{\zeta_0} \tag{33}$$

and for the reradiated wave from (29) to (31),

$$|\bar{S}_r\cdot\hat{n}_r| = \frac{1}{2}\zeta_0\left(\frac{E_0\lambda}{\pi Z_0}\right)^2\frac{F^2}{R^2}G^2, \tag{34}$$

where $F = |\bar{F}|$. We thus obtain

$$\sigma(\theta, \phi) = \frac{4}{\pi}\lambda^2\left(\frac{\zeta_0}{Z_0}\right)^2 F^2(\theta, \phi)G^2(\theta, \phi). \tag{35}$$

D. Numerical Results

In Fig. 3, the results of computations based on the method outlined above are compared with results calculated in a similar way, but with scattering and mutual impedances neglected. The curves are calculated for a four-element linear half-wave dipole reflector with a distance d between adjacent elements equal to 0.2λ,

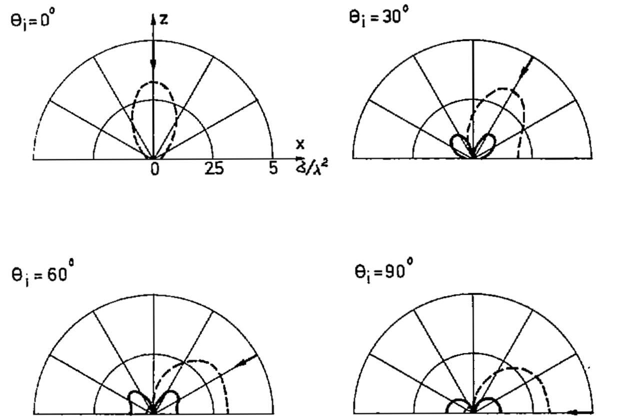


Fig. 3. Relative differential scattering cross section σ/λ^2 as a function of the angle of incidence. $a=p\lambda$, $d=0.2\lambda$, $Z_0=Z_A=75\ \Omega$. Solid line: mutual impedances and scattering taken into account. Dotted line: mutual impedances and scattering neglected. Bold arrows indicate angle of incidence.

$a=p\lambda$, $Z_0=Z_A=75\ \Omega$. The self-and mutual-impedances of the half-wave dipoles have been found from Stearns' table [28].

It is seen that when scattering and mutual coupling are neglected, the reflector shows the performance expected of a Van Atta reflector. However, the results based on the theory given in this paper are quite different. So, for the example shown, there is no reflection at all for normal incidence, and the reradiation pattern is symmetrical with respect to the normal to the reflector. For a further discussion of the results based on the theory outlined here, the reader is referred to Appel-Hansen [29], where numerical results are discussed and compared with experimental results.

IV. Conclusion

A survey of the literature on Van Atta reflectors has been given. A method for a theoretical investigation of such reflectors has been suggested. The method is applicable for an arbitrary Van Atta reflector; however the calculations have only been carried out for a dipole reflector.

A numerical example of a four-element linear reflector shows that the method outlined here—with scattering by the antennas and mutual impedances taken into account—gives results which differ considerably from those found using the simpler theories. However, it is expected that reflectors with other types of antenna elements than dipoles will act more as predicted in the patent description.

Acknowledgment

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